



ICC'18 AHSN

Integrated Localization and Control for Accurate Multi-Agent Formation

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Outline

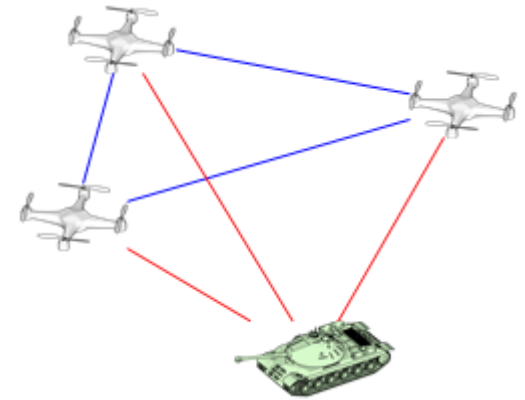
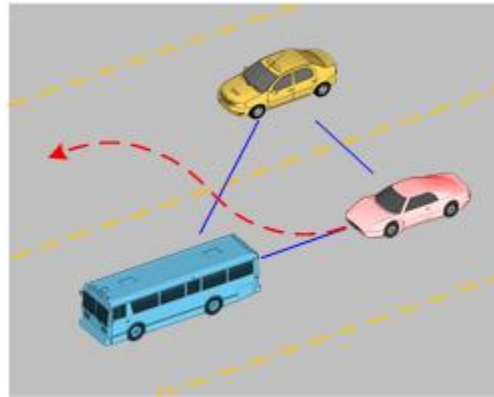
- Introduction
- Performance Metric
- Integrated Scheme
- Numerical Results
- Conclusions

Introduction

- Multi-Agent Systems

- Applications

- Internet of Vehicles
- Target tracking
- Cooperative SLAM
- Communication relaying
- Precise agriculture



High-Accuracy formation is essential for multi-agent systems to accomplish numerous missions!



How to realize high-accuracy formation in *wireless mobile networks*, which is threatened by the *limitation of spectrum resource*?

Introduction

- Existing Methods (*Two Separate Steps*)
 - Network Localization
 - *Analytical framework*: based on *Fisher information*^[1]
 - *Relative localization*: *shape estimation* of the network^[2]
 - *Network scheduling*: selection of the *measurement links*^[3]
 - Formation Control

[1] Y. Shen, H. Wymeersch, and M. Z. Win, “Fundamental limits of wideband localization – Part II: Cooperative networks,” *IEEE Trans. Inf. Theory*, vol. 56, no. 10, pp. 4981–5000, Oct. 2010.

[2] J. N. Ash and R. L. Moses, “On the relative and absolute positioning errors in self-localization systems,” *IEEE Trans. Signal Process.*, vol. 56, no. 11, pp. 5668 – 5679, Jun. 2008.

[3] T. Wang, Y. Shen, A. Conti, and M. Z. Win, “Network navigation with scheduling: Error evolution,” *IEEE Trans. Inf. Theory*, vol. 63, no. 11, pp. 7509–7534, Nov. 2017.

Introduction

- Existing Methods (*Two Separate Steps*)
 - Network Localization
 - Formation Control
 - *Type of observations*: range only, no global coordinate^[1]
 - *Graph-based control*: potential function based method^[2]
 - *Performance evaluation*: stability, rather than accuracy^[3]

[1] K. K. Oh, M. C. Park, and H. S. Ahn, “A survey of multi-agent formation control,” *Automatica.*, vol. 53, no. s, pp. 424–440, Mar. 2015.

[2] M. Cao, C. Yu, and B. D. Anderson, “Formation control using range-only measurements,” *Automatica.*, vol. 47, no. 4, pp. 776–781, Apr. 2011.

[3] Z. Lin, B. Francis, and M. Maggiore, “Necessary and sufficient graphical conditions for formation control of unicycles,” *IEEE Trans. Autom. Control*, vol. 50, no. 1, pp. 121–127, Jan. 2005.

Introduction

- Drawbacks

- Performance Metric

- Either **the probability of convergence** to the target formation, or **the deviation of the true trajectory** from the planned one
 - The absence of a metric to **characterize the accuracy** of the formation

- Separate Scheme

- **Stepwise optimization** may lead to suboptimal performance, since
 - 1) the **information dissipates** between the procedures
 - 2) the **localization-optimal** scheduling strategy may not be **formation-optimal**, especially when the resource is very limited
 - **Cross-step optimization** can improve the overall performance

Introduction

- Motivation

- Performance Metric

- A new metric to **quantize the difference** between the **true formation** and the **target formation**

- Integrated Scheme

- The effect of localization on control: employ the **estimated formation** and the **distribution information** when determining the control vector
(Better utilization of the information in the measurements)
 - The effect of control on localization: take the **target formation** and the **control policy** into consideration when designing scheduling strategy
(Better allocation of the limited spectrum resource)

Introduction

- System Model

- Control Model (Global & Local)

- Suppose the agents can directly control their locations

$$\mathbf{x}_i^{(t)} = \mathbf{x}_i^{(t-1)} + \mathbf{c}_i^{(t)} + \mathbf{w}_i^{(t)}$$

↑
Agent's location

↑
Control error

- Measurement Model (Range)

- Distance measurements of neighboring agents

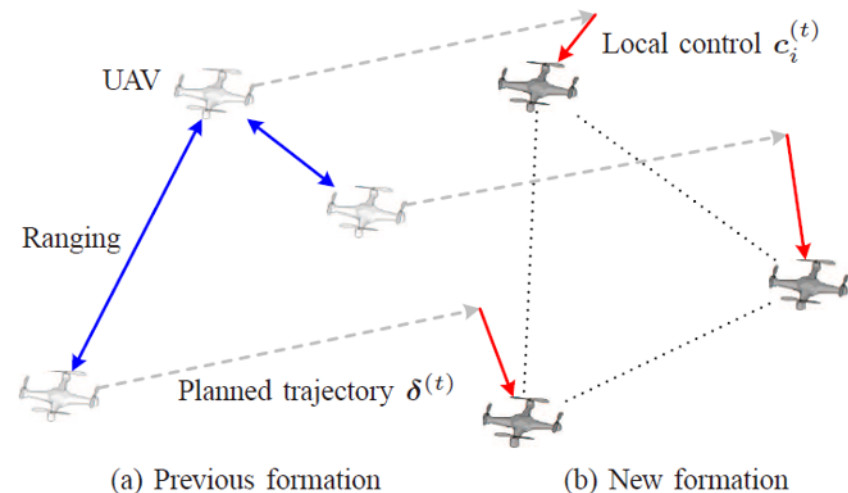
$$d_{ij}^{(t)} = \|\mathbf{x}_i^{(t)} - \mathbf{x}_j^{(t)}\| + v_{ij}^{(t)}$$

↑
Measurement uncertainty

Without loss of generality, we can suppose $\delta^{(t)} = 0$

Formation

$$\mathbf{x} = [\mathbf{x}_1^T \quad \dots \quad \mathbf{x}_N^T]^T$$



Outline

- Introduction
- **Performance Metric**
- Integrated Scheme
- Numerical Results
- Conclusions

Performance Metric

- Difference Between Formations

- Not related with *the position* and *the orientation* of the formation
- Feasible Set
 - The feasible set $\mathcal{S}(\xi)$ of the formation ξ is defined as the collection of the formations that can be achieved by *applying the translation and rotation operations* on formation ξ .
 - Mathematical description

$$\mathcal{S}(\xi) = \{ [\mathbf{I}_N \otimes \mathbf{R}(\vartheta)] \xi + \mathbf{1}_N \otimes \mathbf{t} : \mathbf{t} \in \mathbb{R}^2, \vartheta \in [0, 2\pi) \}$$

$$\mathbf{R}(\vartheta) = \begin{bmatrix} \cos \vartheta & -\sin \vartheta \\ \sin \vartheta & \cos \vartheta \end{bmatrix}$$

Kronecker product

Performance Metric

- Formation Error

- Characterize the *minimum squared distance* between the true formation \mathbf{x} and the target formation ξ *over arbitrary translation and rotation*, denoted by $\mathcal{F}(\mathbf{x}, \xi)$.

- Optimization description

$$\mathcal{F}(\mathbf{x}, \xi) = \min_{\mathbf{s} \in \mathcal{S}(\xi)} \|\mathbf{x} - \mathbf{s}\|^2$$

- Close-form solution

$$\mathcal{F}(\mathbf{x}, \xi) = \|\mathbf{D}\mathbf{x}\|^2 + \|\xi\|^2 - 2 \|\xi\| [\mathbf{x}^T (\mathbf{D}\mathbf{P}\mathbf{D})\mathbf{x}]^{1/2}$$

$$\mathbf{D} = \mathbf{I}_{2N} - \frac{1}{N} (\mathbf{1}_N \mathbf{1}_N^T) \otimes \mathbf{I}_2$$

Translate a formation until its mass center locates at the origin

$$\mathbf{P} = \frac{1}{\|\xi\|^2} (\xi \xi^T + \eta \eta^T)$$

Target formation η that can be achieved by rotating ξ for $\pi/2$

Performance Metric

- Formation Error

- Equivalent Expression

- Define the regularized formation $\mathbf{y} = D\mathbf{x}$
- Decomposed form of the formation error

$$\mathcal{F}(\mathbf{x}, \boldsymbol{\xi}) = \|\mathbf{y}_{\perp}\|^2 + (\|\mathbf{y}_{//}\| - \|\boldsymbol{\xi}\|)^2$$

$$\mathbf{y}_{\perp} = (\mathbf{I} - \mathbf{P})\mathbf{y}$$

$$\mathbf{y}_{//} = \mathbf{P}\mathbf{y}$$

- This equivalent form is derived from the perspective of **subspace theory**, and yields a **geometrical interpretation**.

Performance Metric

- Formation Error

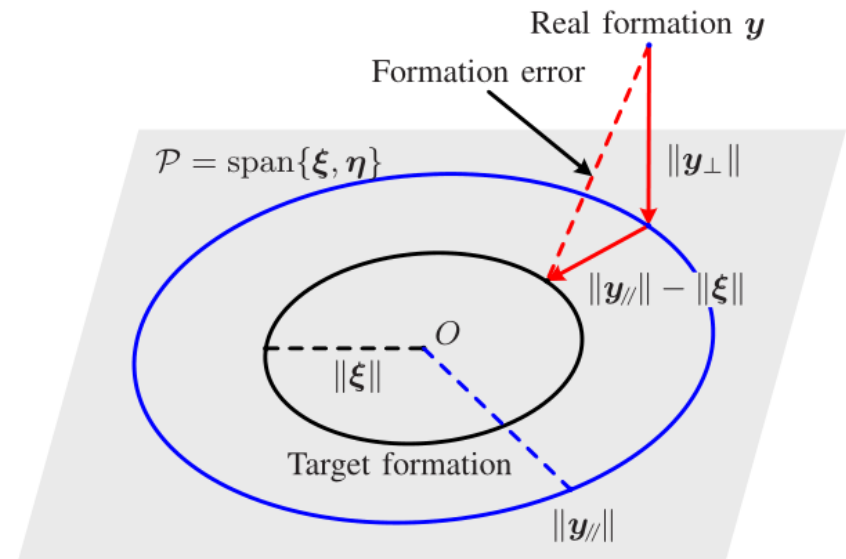
- Geometrical Interpretation

- *Proposition:* The subset of $\mathcal{S}(\xi)$, which includes all target formations with mass centers at the origin, is a circle of radius $\|\xi\|$ centered at the origin in the hyperplane $\mathcal{P} = \text{span}\{\xi, \eta\}$.

- Formation error

$$\mathcal{F}(x, \xi) = \|\mathbf{y}_\perp\|^2 + (\|\mathbf{y}_\parallel\| - \|\xi\|)^2$$

- Parallel Component \mathbf{y}_\parallel
- Parallel Error $(\|\mathbf{y}_\parallel\| - \|\xi\|)^2$
- Orthogonal Component \mathbf{y}_\perp
- Orthogonal Error $\|\mathbf{y}_\perp\|^2$



Outline

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Integrated Scheme

- General Framework

- A typical division of steps in cyber-physical systems

- Environment sensing

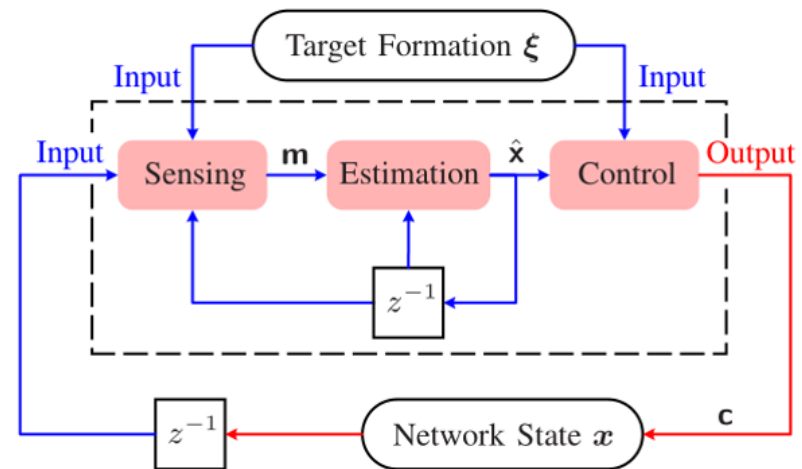
make measurements \mathbf{m} according to certain scheduling strategy

- State estimation

estimate the true formation based on the observations

- Formation control

adjust the formation \mathbf{c} according to certain control policy



Integrated Scheme

- Integrated Localization and Control

- Formation after control

$$\mathbf{x}^{(t)} = \mathbf{x}^{(t-1)} + \mathbf{c}^{(t)} \left(\hat{\mathbf{x}}^{(t)} \left(\mathbf{m}^{(t)} \right) \right) + \mathbf{w}^{(t)}$$

- The **measurement** is determined by the **scheduling strategy** \mathcal{A}

- The **control vector** is determined by the **control policy** \mathcal{C}

- Performance metric *mean formation error*

$$F(\{\mathcal{A}, \mathcal{C}\}) = \mathbb{E}\{\mathcal{F}(\mathbf{x}^{(t)}, \boldsymbol{\xi})\}$$

- *Optimal combination* of the **scheduling strategy** and the **control policy**

$$\{\mathcal{A}, \mathcal{C}\}^* = \arg \min_{\mathcal{A}, \mathcal{C}} F(\{\mathcal{A}, \mathcal{C}\})$$

Integrated Scheme

- Integrated Localization and Control

- Optimization in a **simultaneous manner**

- Manipulate the scheduling strategy \mathcal{A} and the control policy \mathcal{C} **at the same time** to solve the problem
- Not tractable due to the intricate relationship of the two steps

- Optimization in an **iterative manner**

- Given **control policy** \mathcal{C}_0 , the **optimal scheduling strategy** \mathcal{A}^* refers to

$$\mathcal{A}^*(\mathcal{C}_0) = \arg \min_{\mathcal{A}} F(\{\mathcal{A}, \mathcal{C}_0\})$$

- Given **scheduling strategy** \mathcal{A}_0 , the **optimal control policy** \mathcal{C}^* refers to

$$\mathcal{C}^*(\mathcal{A}_0) = \arg \min_{\mathcal{C}} F(\{\mathcal{A}_0, \mathcal{C}\})$$

Integrated Scheme

- Case Study

- Design the **scheduling strategy** under a given **control policy**

- **MMFE control**

- Minimize the **mean formation error** when the *true formation* \mathbf{x} is known, in the presence of control error $\mathbf{w} \sim \mathcal{N}(\mathbf{0}, \sigma_c^2 \mathbf{I})$

$$F = \|\mathbf{y}_\perp + \mathbf{c}_\perp\|^2 + h(\|\mathbf{y}_\parallel + \mathbf{c}_\parallel\|^2) + (2N - 2)\sigma_c^2$$

- The control vectors that minimize the above expression

$$\mathbf{C}^*(\mathbf{x}) = \rho \frac{PD\mathbf{x}}{\|PD\mathbf{x}\|} - D\mathbf{x}$$

- When the true formation is not available, the policy use the estimated formation instead of the true formation to calculate the vector

Integrated Scheme

- Case Study

- Design the **scheduling strategy** under a given **control policy**
- *Scheduling strategy that suits the MMFE control*
 - **Formation after control** is

$$\mathbf{x}^+ = \mathbf{x} + C^*(\hat{\mathbf{x}}) + \mathbf{w}$$

- The **approximate theoretical bound** of the MFE

$$\begin{aligned} F &= \mathbb{E}\{\mathcal{F}(\mathbf{x} + C^*(\hat{\mathbf{x}}) + \mathbf{w}, \boldsymbol{\xi})\} \\ &\approx \mathbb{E}\{(\mathbf{y} - \hat{\mathbf{y}})^T \mathbf{B}(\mathbf{y} - \hat{\mathbf{y}})\} + (\rho - \|\boldsymbol{\xi}\|)^2 + \left(2N - 2 - \frac{\|\boldsymbol{\xi}\|}{\rho}\right) \sigma_c^2 \\ &\geq \text{tr}\{\mathbf{B}\mathbf{D}\mathbf{J}(\mathbf{x})^{-1}\} + (\rho - \|\boldsymbol{\xi}\|)^2 + \left(2N - 2 - \frac{\|\boldsymbol{\xi}\|}{\rho}\right) \sigma_c^2 \end{aligned}$$

Error resulted from the
localization step

Error resulted from the
control step

Integrated Scheme

- Case Study

- Design the **scheduling strategy** under a given **control policy**

- *Scheduling strategy that suits the MMFE control*

- Minimize the **approximate theoretical bound of the MFE**, under the given number L of the measurement links

$$\mathcal{A}^* = \arg \min_{\mathcal{A}} \text{tr}\{\mathbf{BDJ}(\mathbf{x})^{-1}\}$$

- A greedy realization of this algorithm: choose L links one by one, and in each resource unit, select the link with the **largest weight**

$$v_{i,j} = \text{tr}\{\mathbf{BD}[\mathbf{J}_0(\mathbf{x})^{-1} - \mathbf{J}_{i,j}(\mathbf{x})^{-1}]\}$$

which is the **reduction of the approximated MFE bound** if the link is selected

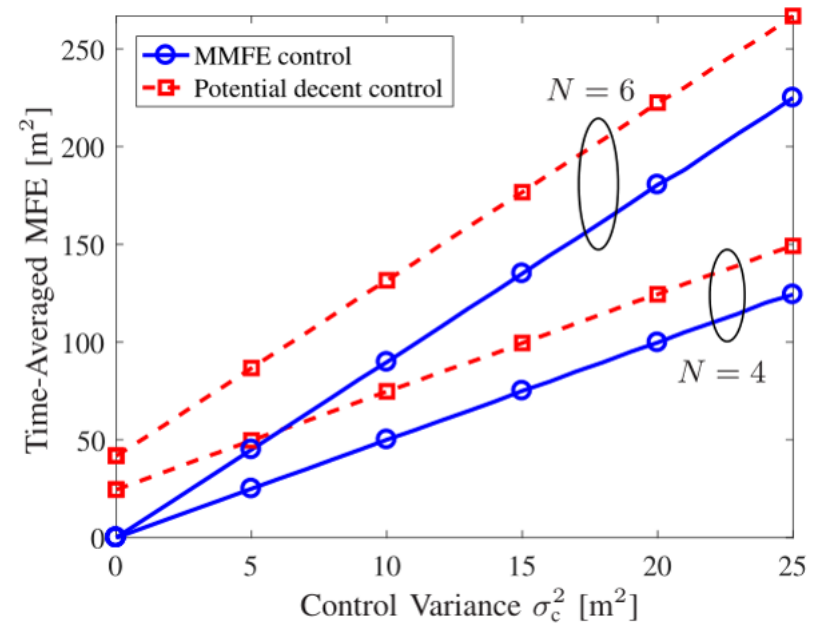
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Numerical Results

- General Configuration
 - A network of N UAVs
 - The target formation is a uniform line
- Control policy
 - Comparison
 - The MMFE control policy
 - The potential decent control
 - Gain of the proposed method
 - Higher accuracy
 - Smaller control cost
 - Lower computational complexity

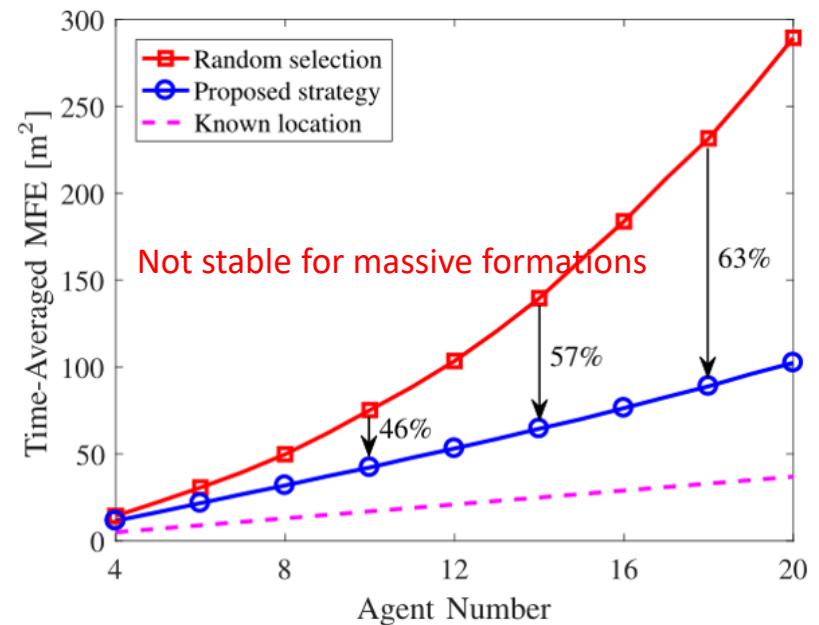
The true formation \mathbf{x} is generated from $\mathcal{N}(\xi, \mathbf{I})$



Numerical Results

- General Configuration
 - A network of N UAVs
 - The target formation is a uniform line
- Scheduling Strategy
 - Comparison
 - The proposed scheduling strategy
 - Random scheduling
 - Known position (baseline)
 - Gain of the proposed method
 - Linear with the agent number
 - The gap grows with agent number

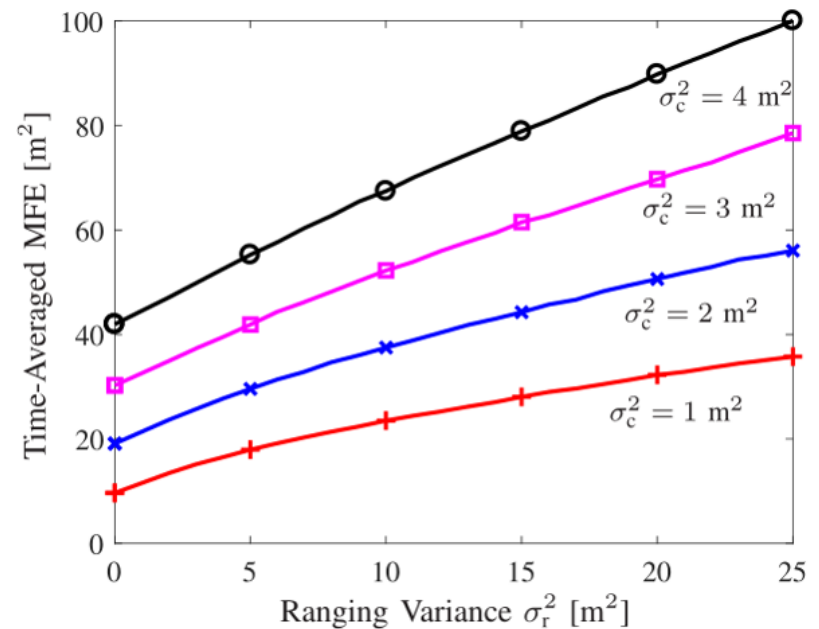
The MMFE control policy is adopted



Numerical Results

- General Configuration
 - A network of 4 UAVs
 - The target formation is a uniform line
- System Parameter
 - Insights
 - Intricate relationship between the MFE and both parameters
 - Analyze the bound of the MFE
 - A special case $\sigma_c \gg \sigma_r$
 - The MFE (or weighted CRB) is approximately linear with both parameters σ_c^2 and σ_r^2

The variance of the ranging noise & the control error



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Conclusions

- A new performance metric
 - Formation error
 - Optimization formulation and a closed-form solution
 - An equivalent expression and the geometrical interpretation
- The integrated localization and control scheme
 - Advantages
 - Better allocation of the resource
 - Better utilization of the information
 - Case study
 - Optimize the scheduling strategy and the control policy iteratively
 - The MMFE control and the scheduling strategy that suits it

Q & A

Thanks for your attention!

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